Dynamic Problem of Covering a Region Bounded by a Curved Line with Circles

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***Abstract* — The article considers the analysis of specific discrete optimization problems taking into account dynamics, when changes in parameters, or the so-called "disturbance" of the system, are to be taken into account. Both single-criteria and multi-criteria cases and their application in various areas of microeconomics are considered. Fundamental issues and algorithms of discrete optimization, dynamic programming and graph theory are used to solve these problems.**

In particular, a multi-parameter dynamic problem of covering a region bounded by a curved line with circular objects with a varying radius is considered. Optimization of the characteristic values of objects is considered as an optimality criterion. The coverage radius, power and spatial parameters of each object are selected as parameters.

A mathematical model and an algorithm of polynomial complexity have been developed for this problem.

***Keywords*—Discrete optimization, plane covering problem, algorithm.**

I. INTRODUCTION

Manufacturing always faces the problem of increasing production efficiency. In this regard, the greatest importance is attached to the tasks of optimal cutting, which arise in a number of industries, such as engineering, shipbuilding, woodworking, glass processing, house building, manufacturing of various equipment, paper production, sewing factories and other areas.

As a rule, the products from which the parts are to be manufactured are supplied in the form of rolls, rods, rectangular plates. From the incoming materials, it is necessary to cut out parts of specified dimensions and a certain configuration. It is necessary to draw up a plan for cutting these materials in such a way that the amount of useless waste is minimal.

Optimal cutting problems also include the placement of heavy loads in the holds of ships or aircraft, the placement of a large number of finished products in warehouses, etc.

The problem of covering any flat figure with circles can be interesting not only from the point of view of mathematics, but also from the point of view of application in many different fields. Solving such problems is often necessary also to solve various issues in the field of economics, technology and services, such as the placement of antennas or other devices and the regulation of their power, the placement of various service centers and the management of their effective functioning, the solution of the problem of optimal packing in a container, etc.

The vast majority of economic and technical problems are based on data that are discrete in nature and very often unstable, which means that the data can change during the planning process. The corresponding models for such problems belong to the class of dynamic models of discrete optimization and dynamic programming and discrete optimization methods can be used to solve them.

In order to determine the regularities of the processes occurring in the systems existing in manufacturing and industry and to manage them, it is necessary to formalize these processes in the form of a mathematical model.

Building a mathematical model is a complex task, because on the one hand, the model must accurately reflect the physical or economic process and, on the other hand, it must be such that its corresponding algorithm allows solving the problem and obtaining results in a realistically possible time. In general, problems of covering any part of the plane with geometric figures are of NP difficulty, therefore it is possible to build an effective algorithm for them only in certain specific cases.

II. PROBLEM FORMULATION

Let Ω be some connected region with area S. It is necessary to place n {s1, s2, . . . , sn} points (devices) on this region so that each point can monitor a circular region centered at a given point, and the radius is variable and varies in the interval Ri $\in$[Rmin ; Rmax], i=1, 2, . . . n. Let such circles be $C_1, C_2, \dots C_n$. Rmin and Rmax. The values of Rmin and Rmax are calculated and depend on the configuration of Ω and the area of S. Since Ω is part of the plane, the centers

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$s_1, s_2, \ldots, s_n$ are characterized by two coordinates $s_i(x_i, y_i)$ i=1, 2, . . . n.

Devices consume energy to monitor a circular area, which is expressed by an increasing function $\psi_i = f(R_i)$ depending on the radius of the corresponding circle. At the same time, each device is characterized by a quality parameter, which is also a decreasing function $\eta_i = g(Ri)$ depending on the radius of the corresponding circle. In addition, each $\eta_i \geq \eta min$, i=1, 2, . . . n, is a quantity bounded from below.

Let us call P the coverage of Ω region such distribution of devices that any point z of Ω is controlled by at least one device, i.e. belongs to any circle C. $z \in \bigcup_{i=1}^{n} C_i$ and $\Omega \subseteq \bigcup_{i=1}^{n} C_i$. We need to find the coordinates si(xi, yi) of the centers of the covering circles of Ω and for each circle determine the radius Ri i=1, 2, . . . n so that the total energy consumed is maximum and at the same time the total quality is maximum as well

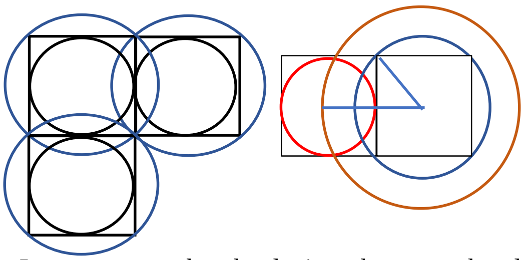
$$\sum_{i=1}^{n} \psi_i \to \max \qquad (1)$$

$$\sum_{i=1}^{n} \eta_i \to \max \qquad (2)$$

Consider the case where the Ω is a rectangle with dimensions a and b, and the area a · b = S. Since n circles are to be placed, we divide the entire area S into n equal parts and say that these parts are squares. Then it is clear that the side of each square will be equal to $\sqrt{\frac{ab}{n}}$. If we place si(xi, yi) i=1, 2, . . . n centers at the centers of the squares, then $R_{min} = \frac{\sqrt{2}}{2}\sqrt{\frac{ab}{n}}$, and $R_{max} = \sqrt{\frac{ab}{n}}$. We introduce the functional

$$\rho = P_1 \sum_{i=1}^{n} \psi_i + P_2 \sum_{i=1}^{n} \eta_i \ , \qquad (3)$$

where $\in[0,1]$ and $P_2 \in [0,1]$. Our goal is to choose Ri $\in$[Rmin; Rmax], i=1,2..n radius such that the ρ functional takes its maximum value. The coefficients P_1 and P_2 are chosen depending on which criterion is given priority at the given moment.



Let us assume that the devices that are to be placed in the centers si(xi, yi), i=1, 2, . . . n,, are of the same type.

III. Algorithm Description

The algorithm for this problem consists of several main steps:

- Divide the connected region Ω into subregions $G_1, G_2, \ldots, G_k$ by reasonable arguments. $\Omega = \bigcup_{i=1}^{k} G_i$.;
- Determine the maximum sizes a_j and b_j, j=1,2, . . . , k, for each set G_j, j=1,2, . . . , k;
- Each region Gj is bounded by a rectangle with area $Sj = a_j \cdot b_j$, the number of objects that must be placed in each region Gj is $n_j = \left[\frac{Sj}{S}\right] n, \ j = 1,2,\ldots,k$;
- Divide the area Sj into n_j parts and consider each of them as a square, the side length of each of which is equal to $\sqrt{\frac{Sj}{n_j}}$and the radius of the circle inscribed in it is equal to $\frac{\sqrt{2}}{2}\sqrt{\frac{Sj}{n_j}}$. It is obvious that the constructed circles cannot cover the area Sj. Therefore, we begin to change the radii and after each change we calculate
$$\rho = P_1 \sum_{i=1}^{n_j} \psi_i(R_j) + P_2 \sum_{i=1}^{n_j} \eta_i(R_j) \ ;$$
- For each subdomain Sj we need to construct an optimal covering scheme, for this we construct a 4-tree for each of the 4 adjacent circles and begin to change the radii, for which we will use the branch and bound method;
- We keep in mind the best value of the functional ρ for each subdomain Sj ρ_j and using the Bellman principle we finally find the best covering scheme for Ω.

For this problem, an algorithm of complexity P with efficiency O(n3) is constructed.

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