Mean-Field Control and Deep Learning for the Information Dissemination Problem in Online Social Networks

Tatiana Zvonareva
Sobolev Institute of Mathematics
Novosibirsk, Russia,
Sirius University,
Sirius, Russia
e-mail: t.a.zvonareva@math.nsc.ru

Olga Krivorotko
Sobolev Institute of Mathematics
Novosibirsk, Russia
Sirius University,
Sirius, Russia
e-mail: o.i.krivorotko@math.nsc.ru

Nikolay Zyatkov
Sobolev Institute of Mathematics
Novosibirsk, Russia
Sirius University,
Sirius, Russia
e-mail: n.y.zyatkov@math.nsc.ru

***Abstract*—The paper formulates and numerically investigates the tasks of stopping and stimulating the dissemination of information in online social networks for a mean-field model and a model based on artificial neural networks. The mean field model is reduced to a joint solution of initial boundary value problems for the Kolmogorov-Fokker-Planck (KFP) and Hamilton-Jacobi-Bellman (HJB) equations, as well as the Nash optimality condition. Numerical methods for solving systems of KFP and HJB in high dimensions are practically nonexistent due to the need for grid-based spatial discretization. Therefore, we propose a deep learning algorithm based on the generative adversarial network. We parameterize the value (the solution of HJB) and density (the solution of KFP) functions by two neural networks that solve a convex-concave saddle-point problem.**

***Keywords*—Mean-field control, deep learning, physics-informed neural networks, Kolmogorov-Fokker-Planck equation, Hamilton-Jacobi-Bellman equation, Nash equilibrium.**

I. Introduction

In the contemporary digital age, online social networks have emerged as powerful platforms, significantly influencing communication patterns, public opinion, and the dissemination of information on a global scale [1]. With the increasing penetration of digital technologies and the ubiquity of social media, understanding and managing the dynamics of information spread has become a subject of pressing importance across multiple domains—including sociology, information science, and applied mathematics. Amidst the many benefits brought by social networks, challenges such as the rapid propagation of unverified information, rumor diffusion, and breaches of privacy have raised concerns about the need for effective control mechanisms.

The process of information dissemination in online social networks can be described by the nonlinear diffusion-logistic equation [2], the coefficients and initial data of which characterize the dissemination of particular information. And adding a control parameter that ensures the Nash equilibrium in the system of interacting agents and minimizes costs, allows one to control the process and transform the problem into a mean-field control [3], [4]. The enormous advantage of the mean field approach is that it allows one to describe the collective behavior of multiple agents making strategic decisions with a small number of equations, which significantly reduces the calculation time and computational complexity. Social networks are no exception in this sense, since the middle-field approach makes it easy to describe the interaction of individuals in a particular population where information is distributed, so they have to make strategic decisions, for example, about involving in the distribution process.

The mean-field problems were numerically investigated in the papers [5]–[8] to describe the dynamics and control of epidemiological and economic processes. Namely, the problem of minimizing the cost function

$$J(u,\alpha) = \int_1^T \int_\Omega F(x, u(x,t), \alpha(x,t))\, dxdt + G(u(x,T))$$

under constraints on the distribution density of agents $u(x,t)$ at the state $x \in \Omega$ at time t that satisfies the Kolmogorov-Fokker-Planck (KFP) equation

$$\begin{cases} \dfrac{\partial u}{\partial t} - D\Delta u + \operatorname{div}\left(u \nabla_p H(x, \nabla v)\right) = 0, \\ u(x,0) = u_0(x), \end{cases}$$

comes down to solving the Hamilton-Jacobi-Bellman (HJB) system

$$\begin{cases} -\dfrac{\partial v}{\partial t} - D\Delta v + H(x, \nabla v) = f(x,u), \\ v(x,T) = G(x, u(x,T)). \end{cases}$$

Here, the function F characterizes the agents' costs for choosing a strategy, G characterizes the terminal cost,

$$H(x,p) = \sup_\alpha [-p \cdot \alpha - L(x,\alpha)]$$

is the Hamiltonian function that depends on the Lagrangian $L(x,\alpha)$, $\alpha(x,t)$ is the control function (representative player's strategy).

The finite-difference schemes have been proposed for the joint solution of systems such as KFP and HJB based on semi-Lagrangian approaches [9].

This study addresses the problem of both stimulating and suppressing information dissemination in online social networks by employing a mean-field modeling approach. Based on the mean-field control theory for describing the dynamics of epidemiology proposed in the research [6], we obtain a system of the KFP and HJB equations for the problem of information dissemination in online social networks.

A similar model was considered in [6], where it was used to simulate the spread of COVID-19 in the Novosibirsk region, which had a population of 2.7 million people in 2021.

The diffusion-logistic model, which underlies the system with the Kolmogorov-Fokker-Planck equation, was validated [2] based on data from the social network Twitter, which had 500 million registered users in 2012.

The calculations were performed on a laptop with an Intel Core i7-11390H processor, 4 cores, a frequency of 3.4 GHz, and 16 GB of RAM.

The model is fundamental in nature, so it is suitable for modelling large social networks. Complicating the model (agent-oriented approach) will necessitate the use of distributed computing.

II. Mean-Field Control for Social Networks

In this section, we formulate control problems aimed at either stopping or stimulating the dissemination of information. They are expressed in terms of cost functionals and are governed by partial differential equations of the KFP and HJB type. The following analysis lays out the mathematical foundation for modeling user behavior and determining optimal control strategies in the context of information flow within social networks.

Let $x \in \Omega = [0,1]$ be understood as the user's state, where 0 means that the user is involved in the information dissemination process, and 1 means that he is not involved. We formulate the problem of stopping the dissemination of information (problem 1)

$$J_1(u_1,\alpha_1) = \int_1^T \int_0^1 \left(d_1 \frac{u_1 \alpha_1^2}{2} + d_2 (x-1)^2 u_1 \right) dxdt \quad (1)$$

and the problem of stimulating the dissemination of information (problem 2)

$$J_2(u_2,\alpha_2) = \int_1^T \int_0^1 \left(d_1 \frac{u_2 \alpha_2^2}{2} + d_2 x^2 u_2 \right) dxdt. \quad (2)$$

Here, d_1 and d_2 are scaling coefficients.

The distribution of users of social network $u_i(x,t)$ in both cases satisfies the system with a KFP-type equation

$$\begin{cases} u_{it} + (u_i\alpha_i)_x + \left(\dfrac{u_i}{K_{cap}} - 1\right) r(t) u_i - D u_{ixx} = 0, \\ u_i(x,1) = u_0(x),\ x \in [0,1], \\ u_{1x}(0,t) - u_i(0,t) = 0,\ u_{ix}(1,t) = 0,\ t \in [1,T], \end{cases} \quad (3)$$

where $i = 1$ for functional (1) and $i = 2$ for functional (2).

Here, the third term (the logistic term) describes the dynamics of the population under consideration and reflects the influence of the structure of a particular social network on the growth of the density of $u(x,t)$. The Robin boundary condition at $x = 0$ reflects the fact that information is exchanged, and the condition at $x = 1$ means that there is no information flow across the boundary.

We consider the parameters of the model that correspond to the data of the news site Digg.com, represented in [2], where $D = 0.01$ is the diffusion coefficient, $K_{cap} = 25$ is the constant describing the maximum network bandwidth, and $r(t) = 1.4e^{-1.5(t-1)} + 0.25$ is the growth rate of the number of active users.

To determine the optimal strategies $\alpha_i(x,t)$ for the corresponding functionals J_i, $i = 1,2$, we use the Lagrange multiplier method [5], which formulates the Lagrangians

$$\mathcal{L}_i(u_i,\alpha_i,v_i) = J_i(u_i,\alpha_i) - \int_1^T \int_0^1 \Big[u_{it} + (u_i\alpha_i)_x + \\ + \left(\frac{u_i}{K_{cap}} - 1\right) r(t) u_i - D u_{ixx} \Big] v_i \, dxdt.$$

Differentiating $\mathcal{L}_i$ with respect to u_i, we obtain systems with HJB-type equations

$$\begin{cases} v_{it} + \alpha_i v_{ix} + \left(1 - \dfrac{2u_i}{K_{cap}}\right) r(t) v_i + D v_{ixx} = \\ \qquad = -d_1 \dfrac{\alpha_i^2}{2} - d_2 f_i(x), \\ v_i(x,T) = 0, \quad x \in [0,1], \\ D v_{ix}(0,t) + (\alpha_i(0,t) - D)\, v_i(0,t) = 0, \\ \qquad v_{ix}(1,t) = 0, \quad t \in [1,T]. \end{cases} \quad (4)$$

Here, $f_1(x) = (x-1)^2$ and $f_2(x) = x^2$.

And differentiating $\mathcal{L}_i$ with respect to α_i, we obtain the optimality condition

$$\begin{cases} d_1\alpha_i + v_{ix} = 0, \quad x \in [0,1],\ t \in [1,T], \\ \alpha_i(1,t) = 0, \quad t \in [1,T]. \end{cases} \quad (5)$$

The solutions of the systems (3), (4), (5) provide the necessary conditions for solving the corresponding minimization problems (1) for $i = 1$ and (2) for $i = 2$ and called a mean-field control problem. The solution of the mean-field control problem (3)-(5) consists in determination of user's density $u(x,t)$, individual strategy $v(x,t)$ and the optimality condition $\alpha(x,t)$.

III. Artificial Neural Network

Various approaches using neural networks can be used to solve partial differential equations. For example, [10] describes a numerical method for solving partial differential equations based on a multilayer feedforward neural network. A solver based on a radial basis function neural network was presented in the study [11]. And in [12], physics-informed neural networks (PINN) were used to model heat transfer. The use of neural operators to approximate partial differential equation solvers was also investigated [13].

PINN demonstrates versatility and better integration of the physics of the problem, but can be computationally expensive and sometimes unstable, which can be controlled by choosing a regularization factor in the loss function, choosing a metric, and comparing with classical collocation methods [14].

Rather than discretizing the domain and solving for the function values at grid-points, an artificial neural network avoids them by parameterizing the function and solving for the function itself [15]–[18]. By using training paradigms inspired by generative adversarial networks, it is now possible to compute optimal control strategies in large and realistic social network scenarios efficiently. To do this, the neural networks N_ω and N_θ with weights ω and θ are initialized, and solutions of HJB and KFP are introduced in the form

$$\Phi(x,t) = (T-t+1)N_\omega(x,t),$$
$$\Gamma(y,t) = (T-t+1)y + (t-1)N_\theta(y,t),$$

where $y \sim u_0$ are samples drawn from the initial distribution. In this setting, we train $\Gamma(\cdot,t)$ to produce samples from $u(\cdot,t)$. We note that Φ and Γ automatically satisfy the terminal (zero in our functions (1) and (2)) and initial conditions, respectively. Our strategy for training consists of alternately training Γ (the population) and Φ (the value function for an individual agent) close to the GAN approach [19]. Let us omit the index i in this section.

The training process begins with the following steps:

To train the network Φ (discriminator):

1) Sample batch $\{(y_b,t_b)\}_{b=1}^B$ with batch size B, where $y_b \sim u_0$, $t_b \sim U(1,T)$ (uniform distribution).

2) Compute the push-forward states $x_b = \Gamma(y_b,t_b)$.

3) Minimize the loss function with a regularization term to penalize deviations from the HJB equation

$$l_\Phi = \frac{1}{B}\sum_{b=1}^{B}\Bigg[\Phi(x_b,1) + \Phi_t(x_b,t_b) + D\Phi_{xx}(x_b,t_b)+ +\alpha(x_b,t_b)\Phi_x(x_b,t_b) + \left(1-\frac{2u}{K_{cap}}\right)r(t_b)\Phi(x_b,t_b)\Bigg] + + \lambda\frac{1}{B}\sum_{b=1}^{B}\Bigg\|\Phi_t(x_b,t_b) + D\Phi_{xx}(x_b,t_b) + \alpha(x_b,t_b)\Phi_x(x_b,t_b) + \left(1-\frac{2u}{K_{cap}}\right)r(t_b)\Phi(x_b,t_b) - d_1\frac{\alpha^2(x_b,t_b)}{2} - d_2 f(x)\Bigg\|$$

and find the weights ω of the network Φ.

To train the network Γ (generator)

4) Sample batch $\{(y_b,t_b)\}_{b=1}^B$, where $y_b \sim u_0$, $t_b \sim U(1,T)$.

5) Minimize the loss function

$$l_\Gamma = \frac{1}{B}\sum_{b=1}^{B}\Bigg[\Phi_t(\Gamma(y_b,t_b),t_b) + D\Phi_{xx}(\Gamma(y_b,t_b),t_b)+ + \alpha(\Gamma(y_b,t_b),t_b)\Phi_x(\Gamma(y_b,t_b),t_b)+ + \left(1-\frac{2u}{K_{cap}}\right)r(t_b)\Phi(\Gamma(y_b,t_b),t_b) - -d_1\frac{\alpha^2(\Gamma(y_b,t_b),t_b)}{2} - d_2 f(x)\Bigg]$$

and find the weights θ of the network Γ.

And the stopping criterion is the number of iterations (epochs).

IV. Numerical Experiments

The problems (3), (4), (5) were solved by a finite difference scheme suggested in [9].

The values of the functions u_i and v_i were calculated on uniform grids with respect to time $t_l = l\tau$, $l = 0,\dots,N_t$, $\tau = (T-1)/N_t$ and space $x_{j+1/2} = (j+1/2)h$, $j = -1,\dots,N_x$, $h = 1/N_x$. A scheme from [6] with an approximation order of $O(\tau + h^2)$ was used, where the left boundary conditions were approximated as follows:

$$u_{i;l,-1/2} = \frac{2-h}{2+h}u_{i;l,1/2},$$
$$v_{i;l,-1/2} = \frac{D/h - (D-\alpha_{i;l,0})/2}{D/h + (D-\alpha_{i;l,0})/2}v_{i;l,1/2}.$$

The values of the function α, unlike the functions u_i and v_i, were calculated on a uniform grid over the space $x_j = jh$, $j = 0,\dots,N_x$, $h = 1/N_x$.

In numerical experiments, we assume $T = 24$, $N_x = 30$, $N_t = 1500$, $d_1 = 100$, $d_2 = 1$. Such values were chosen to satisfy the limitations of the method [6]

$$h^2 \le 8\tau D \quad \text{and} \quad \tau|\alpha_{l,j}| \le h/4. \tag{6}$$

And the initial user density function $u_0(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d are determined from the vector of values $(5.8, 1.7, 1.9, 1, 0.95, 0.7)$ at $x = 1,2,3,4,5,6$ using spline interpolation approach.

Figure 1 shows graphs of the distribution of users by degree of involvement $u_i(x,t)$ of the model with an equation of the KFP type (3), cost functions $v_i(x,t)$ of models with an equation of the HJB type (4) and the control functions $\alpha_i(x,t)$ satisfying the optimality condition (5).

We can see that when solving the assigned synthetic problems, the optimal values of these functions for the corresponding functionals are determined, and the set goals of stopping and stimulating the dissemination of information are achieved. Indeed, the first row of Fig. 1 at the final moment of time $t = 24$ hours, the distribution density of users $u_1(x,t)$ is concentrated in the vicinity of $x = 1$, which shows that the strategy $\alpha_1(x,t)$ of limiting the dissemination of information is effective. In the case of stimulating the dissemination of information (the second row of Fig. 1), on the contrary, the distribution density of involved users $u_2(x,t)$ is concentrated in the vicinity of $x = 0$.

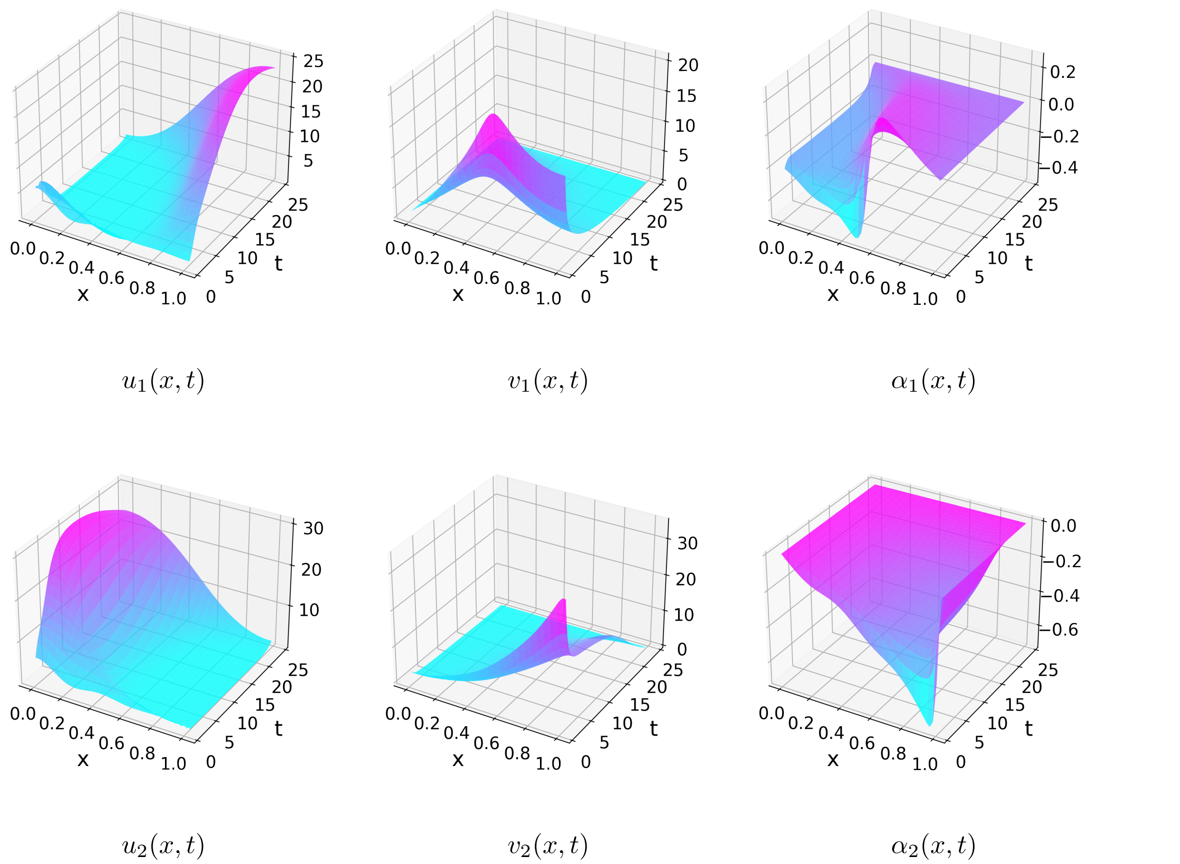

Fig. 1. Functions $u_i(x,t)$, $v_i(x,t)$ and $\alpha_i(x,t)$ of the mean-field models, where $i = 1$ for functional (1) (suppressing information) and $i = 2$ for functional (2) (stimulating information)

V. Conclusion and Future Work

In this study, we developed and numerically investigated mean-field control models describing both the suppression and stimulation of information dissemination in online social networks. We demonstrated that deep neural network algorithms, which approximate solutions to high-dimensional Kolmogorov-Fokker-Planck and Hamilton-Jacobi-Bellman equations, are highly promising for overcoming the computational limitations inherent in traditional grid-based methods. Our approach enables the efficient computation of optimal strategies for information flow management, which is of practical importance for digital platform governance. Optimal strategies for the behavior of synthetic users involved in the information dissemination process have been identified for these cases.

Future research will focus on several key directions:

- Application to real-world historical data (Twitter, Facebook, etc.);
- Focus on automated hyperparameter selection and integrating the developed neural network algorithms with streaming data for real-time monitoring and dynamic control in social networks;
- We will explore multipopulation and hierarchical mean-field models that account for user heterogeneity and platform-specific features, broadening the practical relevance of our approach.

Acknowledgment

This work was supported by the grant of the state program of the “Sirius” Federal Territory “Scientific and technological development of the “Sirius” Federal Territory” (Agreement no. 26-03 date 07.07.2025).

References

[1] S. Sawyer and M. Tyworth, “Social Informatics: Principles, Theory, and Practice”, *Proceedings of the Seventh International Conference on Human Choice and Computers*, Maribor, Slovenia, pp. 49–62, 2006.

[2] F. Wang, H. Wang and K. Xu, “Diffusive Logistic Model Towards Predicting Information Diffusion in Online Social Networks”, *Proceedings of the International Conference on Distributed Computing Systems Workshops*, Macau, China, pp. 133–139, 2012.

[3] J.M., Lasry and P.L. Lions, "Mean field games", *Japanese Journal of Mathematics*, vol. 2, pp. 229–260, 2007.
[4] T. A. Zvonareva and O. I. Krivorotko, "Modeling of the information dissemination in online social networks based on mean-field approach", *Siberian Electronic Mathematical Reports*, vol. 22, no. 1, pp. 307–325, 2025.
[5] A. Bensoussan, J. Frehse and P. Yam, *Mean Field Games and Mean Field Type Control Theory*, Springer, New York, 2013.
[6] V. Petrakova and O. Krivorotko, "Mean field game for modeling of COVID-19 spread", *Journal of Mathematical Analysis and Applications*, vol. 514, pp. 126271, 2022.
[7] Y. Achdou, P. Cardaliaguet, F. Delarue, A. Porretta and F. Santambrogio, *Mean Field Games*, Springer, Cham, 2020.
[8] A. Lachapelle, J.-M. Lasry, C.-A. Lehalle and P.-L. Lions, "Efficiency of the price formation process in presence of high frequency participants: a mean field game analysis", *Mathematics and Financial Economics*, vol. 10, pp. 223–262, 2016.
[9] V. Shaydurov and V. Kornienko, "A finite-difference solution of mean field problem with a predefined control resource", *AIP Conference Proceedings*, Albena, Bulgaria, pp. 110004, 2020.
[10] M.W.M.G. Dissanayake and N. Phan-Thien, "Neural-network-based approximations for solving partial differential equations", *Communications in Numerical Methods in Engineering*, vol. 10, no. 3, pp. 195–201, 1994.
[11] A. Tolstykh and D. Shirobokov, "On using radial basis functions in a "finite difference mode" with applications to elasticity problems", *Computational Mechanics*, vol. 33, pp. 68–79, 2003.
[12] S. Cai, Z. Wang, S. Wang, P. Perdikaris and G. E. Karniadakis, "Physics-informed neural networks for heat transfer problems", *Journal of Heat Transfer*, vol. 143, no. 6, pp. 060801, 2021.
[13] N. Kovachki, Z. Li, B. Liu, K. Azizzadenesheli, K. Bhattacharya, A. Stuart and A. Anandkumar, "Neural Operator: Learning Maps Between Function Spaces With Applications to PDEs", *Journal of Machine Learning Research*, vol. 24, no. 89, pp. 1–97, 2023.
[14] A. Neverov and O. Krivorotko, "Neural nets for modelling of scenarios and control of epidemics", *Proceedings of the First Conference of Mathematics of AI*, 2025.
[15] E. Haroutunian, M. Haroutunian and N. Afshar, "Random coding bound for E-capacity of the wiretap channel", *Proceedings of International Conference Computer Science and Information Technologies*, pp. 121–124, 2011.
[16] L. Ruthotto,S.J. Osher,W. Li,L. Nurbekyan and S.W. Fung, "A machine learning framework for solving high-dimensional mean field game and mean field control problems", *Proceedings of the National Academy of Sciences of the United States of America*, vol. 117, no. 17, pp. 9183–9193, 2020.
[17] A.T. Lin, S.W. Fung, W. Li, L. Nurbekyan and S.J. Osher, "Alternating the population and control neural networks to solve high-dimensional stochastic mean-field games", *Proceedings of the National Academy of Sciences of the United States of America*, vol. 118, no. 31, pp. e2024713118, 2021.
[18] G. Wang, J. Fang, L. Jiang, W. Yao and N. Li, "Coupled Alternating Neural Networks for Solving Multi-Population High-Dimensional Mean-Field Games", *Mathematics*, vol. 12, no. 23, pp. 3803, 2024.
[19] I.J. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville and Y. Bengio, "Generative adversarial nets", *Proceedings of the 27th International Conference on Neural Information Processing Systems*, vol. 2, pp. 2672–2680, 2014.