A new iterative algorithm for computation of the real stability radius

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***Abstract*—This paper provides a new simple method for computing the real stability radius in the Frobenius norm and corresponding destabilizing perturbation. The method is based on Givens rotations and does not require any initial guess.**

***Keywords*—Destabilizing perturbation, real stability radius, distance to instability, Frobenius norm.**

I. Introduction

The matrix $A \in \mathbb{R}^{n\times n}$ is called **stable** (**Routh – Hurwitz stable**) if all its eigenvalues are situated in the open left half plane of the complex plane. For a stable matrix A, some perturbation $E \in \mathbb{R}^{n\times n}$ may lead to that eigenvalues of $A+E$ cross the imaginary axis, i.e., to loss of stability. Given some norm $||\cdot||$ in $\mathbb{R}^{n\times n}$, the smallest perturbation E that makes $A+E$ unstable is called the **destabilizing real perturbation**. It is connected with the notion of the **distance to instability (stability radius) under real perturbations** that is formally defined as

$$\beta_{\mathbb{R}}(A) = \min\{||E|| \,\big|\, \eta(A+E) \ge 0, E \in \mathbb{R}^{n\times n}\}. \quad (1)$$

Here, $\eta(\cdot)$ denotes the **spectral abscissa** of the matrix, i.e., the maximal real part of its eigenvalues.

The real stability radius problem is important in engineering applications where the dynamics matrix A and its perturbations are typically real. We consider the Frobenius norm that could be easily calculated and is more applicable to the problems arising in Control Theory [3]. There are different approaches to solving the real stability radius problem in the Frobenius norm and in the 2-norm. The 2-norm variant of the problem and the application of pseudospectrum to its solution have been explored intensively ([4], [5], [6], [9], [10]), and the algorithms are based on solving singular value problem. In [2], [13], [14] some lower bounds on the real stability radius in the Frobenius norm were provided. For this case, only a few works present iterative algorithms ([6], [7], [8], [12], [15]). All of these approaches are sensitive to the choice of an initial guess, i.e., the convergence to the true value of the real stability radius cannot be guaranteed. In addition, it is quite impossible to evaluate the computational complexity of known algorithms. It is claimed only that they are quadratically convergent.

In this paper, we propose a new simple iterative method that does not require any initial guess. We prove that the method gives us a local minimum. However, all the conducted numerical experiments result in the true value of the stability radius. For matrices of small sizes ([6], [10], [14]), the values of the real stability radius obtained by the method coincide with those calculated earlier or correspond to the bounds established earlier. In every example, we have also found the corresponding perturbation E and the matrix $B = A+E$ that has an eigenvalue with zero real part. From experimental data, one can assume that the number of iterations depends on

$$\max_{j\in\{1,3,\ldots,n\}} \frac{a_{1j}}{a_{kj}} \quad \text{and} \quad \min_{j\in\{1,3,\ldots,n\}} \frac{a_{1j}}{a_{kj}},$$
$$\max_{j\in\{2,3,\ldots,n\}} \frac{a_{2j}}{a_{kj}} \quad \text{and} \quad \min_{j\in\{2,3,\ldots,n\}} \frac{a_{1j}}{a_{kj}},$$
$$(a_{kj} \ne 0, \qquad k \in \{3,4,\ldots,n\}).$$

If for all rows k these maximal and minimal values are of the same sign and sufficiently close to each other, then the number of iterations is quite small.

II. Preliminaries

We first recall the structure of the manifold in the matrix space that bounds the set of stable matrices. Let $M = [m_{jk}]_{j,k=1}^n \in \mathbb{R}^{n\times n}$ be an arbitrary matrix and

$$f(z) = \det(zI - M) = z^n + a_1 z^{n-1} + \ldots + a_n \in \mathbb{R}^n \quad (2)$$

be its characteristic polynomial. Find the real and imaginary part of $f(x+\mathbf{i}y)$ ($\{x,y\} \subset \mathbb{R}$):

$$f(z) = f(x+\mathbf{i}y) = \Phi(x,y^2) + \mathbf{i}y\Psi(x,y^2)$$

where

$$\begin{aligned} \Phi(x,Y) &= f(x) - \frac{1}{2!}f''(x)Y + \frac{1}{4!}f^{(4)}(x)Y^2 - \ldots, \\ \Psi(x,Y) &= f'(x) - \frac{1}{3!}f^{(3)}(x)Y + \frac{1}{5!}f^{(5)}(x)Y^2 - \ldots \end{aligned}$$

Calculate the resultant of the polynomials $\Phi(0,Y)$ and $\Psi(0,Y)$ in terms of the coefficients of (2):

$$\begin{aligned} K(f) &= \mathcal{R}_Y(\Phi(0,Y), \Psi(0,Y)) \\ &= \mathcal{R}_Y(a_n - a_{n-2}Y + a_{n-4}Y^2 + \ldots, \\ &\qquad a_{n-1} - a_{n-3}Y + a_{n-5}Y^2 + \ldots). \end{aligned} \quad (3)$$

The polynomial $f(z)$ has a root with zero real part iff either $a_n = 0$ or $K(f) = 0$. This results in the following statement [10].

Theorem 1: Equations

$$\det M = 0 \tag{4}$$

and

$$K(f) = \mathcal{R}_Y(\Phi(0,Y), \Psi(0,Y)) = 0 \tag{5}$$

define implicit manifolds in $\mathbb{R}^{n^2}$ that compose the boundary for the **domain of stability**, i.e., the domain in the matrix space $\mathbb{R}^{n\times n}$

$$\mathbb{P} = \{M \in \mathbb{R}^{n\times n} | M \text{ is stable}\}. \tag{6}$$

Therefore, the distance to instability from a stable matrix A is computed as the minimum of the distances to the two algebraic manifolds in $\mathbb{R}^{n\times n}$. The Euclidean distance (Frobenius norm) to the set of singular matrices equals the minimal singular value $\sigma_{\min}(A)$ of the matrix A. The problem of finding the distance to the manifold (5) is much more complicated. Here, we consider one possible approach to its solution.

Remark. Henceforth, we deal solely with the variant of the problem connected with the distance to the manifold (5). In all the results below the value $\beta_{\mathbb{R}}(A)$ is supposed to be achieved in this manifold and the condition $\beta_{\mathbb{R}}(A) < \sigma_{\min}(A)$ is fulfilled. Examples of Section V fall into this case.

Consider Givens rotation that is a rotation in the plane spanned by a pair of coordinate axes. A Givens rotation in the coordinate plane (e_i, e_j) by an angle α clockwise is represented by an orthogonal matrix of the form

$$P_\alpha^{ij} = \begin{bmatrix} 1 & \dots & 0 & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \dots & \cos\alpha & \dots & \sin\alpha & \dots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \dots & -\sin\alpha & \dots & \cos\alpha & \dots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \dots & 0 & \dots & 0 & \dots & 1 \end{bmatrix}.$$

Now consider a matrix $A \in \mathbb{R}^{n\times n}$ and an orthonormal basis $\{e_1, e_2, \dots, e_n\}$. Let us make a rotation in the coordinate plane (e_i, e_j) by an angle α counterclockwise and consider the new matrix: $\tilde{A} = P_\alpha^{ij} A (P_\alpha^{ij})^\top$. In this case, only the entries of matrix A standing in the ith and the jth rows and in the ith and the jth columns are modified:

$$\begin{aligned} &\tilde{a}_{ii} = a_{ii}\cos^2\alpha - a_{ji}\sin\alpha\cos\alpha - a_{ij}\sin\alpha\cos\alpha + a_{jj}\sin^2\alpha, \\ &\tilde{a}_{ji} = a_{ii}\sin\alpha\cos\alpha + a_{ji}\cos^2\alpha - a_{ij}\sin^2\alpha - a_{jj}\sin\alpha\cos\alpha, \\ &\tilde{a}_{ij} = a_{ii}\cos\alpha\sin\alpha - a_{ji}\sin^2\alpha + a_{ij}\cos^2\alpha - a_{jj}\sin\alpha\cos\alpha, \\ &\tilde{a}_{jj} = a_{ii}\sin^2\alpha + a_{ji}\sin\alpha\cos\alpha + a_{ij}\sin\alpha\cos\alpha + a_{jj}\cos^2\alpha, \\ &\tilde{a}_{ik} = a_{ik}\cos\alpha - a_{jk}\sin\alpha, \tilde{a}_{jk} = a_{ik}\sin\alpha + a_{jk}\cos\alpha, \\ &\quad (k \notin \{i,j\}), \\ &\tilde{a}_{ki} = a_{ki}\cos\alpha - a_{kj}\sin\alpha, \tilde{a}_{kj} = a_{ki}\sin\alpha + a_{kj}\cos\alpha, \\ &\quad (k \notin \{i,j\}), \end{aligned} \tag{7}$$

for all others r, q we have $\tilde{a}_{rq} = a_{rq}$.

III. Main Result

In this section, we will use some results that were proved earlier. Here is the lemma of [10].

Lemma 1: Let the nonzero vectors X, Y be orthogonal and $X^\top AY \neq -Y^\top AX$. Then for φ defined by the relations

$$\operatorname{tg} 2\varphi = \frac{X^\top AX - Y^\top AY}{X^\top AY + Y^\top AX}, \ \varphi \in [0, 2\pi), \tag{8}$$

the vectors

$$X' = X\cos\varphi - Y\sin\varphi, \ Y' = X\sin\varphi + Y\cos\varphi \tag{9}$$

satisfy the condition

$$X'^\top AX' = Y'^\top AY' = \frac{1}{2}\left(X^\top AX + Y^\top AY\right). \tag{10}$$

The following theorem was proved in [11].

Theorem 2: Let $A \in \mathbb{R}^{n\times n}$ be a stable matrix, E_* and $B_* = A + E_*$ be the destabilizing perturbation and the nearest to A matrix in the manifold (5) corresponingly. There exists an orthogonal matrix $P \in \mathbb{R}^{n\times n}$ such that the matrix $PB_*P^\top$ is of the lower quasi-triangular form while E_* is a rank 2 matrix with a double real eigenvalue λ_*:

$$E_* = P^\top \left[\begin{array}{ccccc} \lambda_* & 0 & \varepsilon_{13} & \dots & \varepsilon_{1n} \\ 0 & \lambda_* & \varepsilon_{23} & \dots & \varepsilon_{2n} \\ \hline & & \mathbb{O}_{(n-2)\times n} & & \end{array}\right] P. \tag{11}$$

Corollary 1: For the matrix A, we have

$$PAP^\top = \left[\begin{array}{ccccc} -\lambda_* & d_{12} & -\varepsilon_{13} & \dots & -\varepsilon_{1n} \\ d_{21} & -\lambda_* & -\varepsilon_{23} & \dots & -\varepsilon_{2n} \\ \hline & & \mathbb{A}_{(n-2)\times n} & \dots & \end{array}\right],$$

where $d_{12}d_{21} < 0$.

Now let us reformulate the problem of the real stability radius computation with the aid of Lemma 1 and Theorem 2.

PROBLEM. For a given matrix $A \in \mathbb{R}^{n\times n}$, find an orthogonal matrix P such that the sum

$$F(P) = \tilde{a}_{11}^2 + \tilde{a}_{22}^2 + \sum_{k=3}^n \tilde{a}_{1k}^2 + \sum_{k=3}^n \tilde{a}_{2k}^2$$

of entries of the matrix $\tilde{A} = PAP^\top$ is minimal.

This minimum is equal to $[\beta_{\mathbb{R}}(A)]^2$, and matrix P allows us to find the destabilizing perturbation by formula (11).

Remark. It is evident that such orthogonal matrix P is not unique. Really, $\beta_{\mathbb{R}}(A)$ is invariant under any orthogonal transformation of the vector space spanned by $\{e_3, e_4, \dots, e_n\}$.

To solve the problem stated, we need the following lemma.

Lemma 2:

$$a\cos^2\alpha + b\sin^2\alpha = \frac{a+b}{2} + \frac{a-b}{2}\cos 2\alpha.$$

It is well known that every n-dimensional rotation could be represented as a composition of two-dimensional rotations in coordinate planes [1], [16]. In our case, it suffices to consider rotations in coordinate planes (e_1, e_k) and (e_2, e_k), $k \geq 3$.

Hence, we will try to find a sequence of rotations in coordinate planes that converges to the value of real stability radius. It follows that we have to find the minimum value of the function $F(P_\alpha^{ij})$.

Theorem 3: Function $F(P_\alpha^{rk})$ ($r \in \{1,2\}$, $k \geq 3$), treated as a function of α, has its minimum at $\alpha = \alpha_*$ such that

$$\mathrm{tg}\, 2\alpha_* = \frac{2 \sum_{\ell=1,\ell\neq r}^{n} a_{k\ell}a_{j\ell} - 2a_{rk}a_{rj}}{\sum_{\ell=1,\ell\neq r}^{n} (a_{j\ell}^2 - a_{k\ell}^2) + (a_{rk}^2 - a_{rj}^2)}.$$

Proof: The proof of the theorem is by direct calculation of $\frac{d}{d\alpha}F(P_\alpha^{pk})$ with the help of Lemma 2. □

Corollary 2: The function $F(P_\alpha^{rk})$, treated as a function of α, is $(\pi/2)$-periodic. In any interval $[\gamma, \gamma+\pi/2), \gamma \in \mathbb{R}$, this function has a unique maximum and a unique minimum. The stationary values corresponding to the minimum and the nearest maximum differ by $\pi/4$.

To find minimum points, consider the sign of $\frac{d^2}{d\alpha^2}F(P_\alpha^{rk})$.

The function $F(P)$ could be represented as $F(P) = S_1(P) + S_2(P)$ where

$$S_1(P) = \tilde{a}_{11}^2 + \tilde{a}_{22}^2, S_2(P) = \sum_{k=3}^{n} \tilde{a}_{1k}^2 + \sum_{k=3}^{n} \tilde{a}_{2k}^2.$$

In Section IV, we consider two types of rotations in coordinate planes. By rotations in the coordinate plane (e_1, e_2), we maximally diminish the sum S_1 while S_2 remains constant. In this case, we take into account that the sum $\tilde{a}_{11} + \tilde{a}_{22}$ does not change. By rotations in the planes (e_1, e_k) and (e_2, e_k) ($k \geq 3$), we consecutively decrease $F(P)$ regardless the sum S_1.

The rotations of these two types are repeated in sequence until $F(P)$ stops decreasing. Since $F \geq 0$, the obtained decreasing sequence of the corresponding values $F(P)$ converges. Its limit gives us the local minimum of $F(P)$. The limitary orthogonal matrix P can be represented as the product of the rotational matrices P_α^{rk}.

HYPOTHESIS. The sequence of values $F(P_\alpha^{rk})$ corresponding to minimizing rotations in coordinate planes converges to the global minimum that is equal to $[\beta_{\mathbb{R}}(A)]^2$.

Lemma 3: For any rotation in coordinate plane (e_1, e_k) ($k \geq 3$), the values $\tilde{a}_{12}$ and $\tilde{a}_{21}$ do not alter their signs provided that

$$|\mathrm{tg}\,\alpha| < \min\{|a_{12}/a_{k2}|, |a_{21}/a_{2k}|\}.$$

For any rotation in coordinate plane (e_2, e_k) ($k \geq 3$), the values $\tilde{a}_{12}$ and $\tilde{a}_{21}$ do not alter their signs provided that

$$|\mathrm{tg}\,\alpha| < \min\{|a_{12}/a_{1k}|, |a_{21}/a_{k1}|\}.$$

Proof: immediately follows from the formulas (7). □

Thus, for sufficiently small angles of rotation, the inequality $\tilde{a}_{12}\tilde{a}_{21} < 0$ is fulfilled due to Corollary 1.

For the values $\tilde{a}_{12}$ and $\tilde{a}_{21}$ to preserve their signs under processing, we change the rows and columns with numbers i and j ($i \neq j$) such that $a_{ij}a_{ji} < 0$ and the sum $a_{ij}^2 + a_{ji}^2$ is maximal. Then we interchange the first and the ith rows and the first and the ith columns; the second and the jth rows and the second and the jth columns.

IV. ALGORITHM

Input: a stable matrix $A \in \mathbb{R}^{n\times n}$, $\varepsilon > 0$.
Output: real stability radius of A, destabilizing real perturbation.

Procedure 1. Input: matrix $A \in \mathbb{R}^{n\times n}$.
Output: rotation angle

$$\alpha = \mathrm{arctg}\left(\frac{a_{11} - a_{22}}{a_{12} + a_{21}}\right);$$

and rotation matrix P_α^{12} in the plane (e_1, e_2). $\tilde{A} = P_\alpha^{12} A \left(P_\alpha^{12}\right)^\top$;
Set $A = \tilde{A}$;

Procedure 2. Input: matrix $A \in \mathbb{R}^{n\times n}$.
Output: rotation angle α and rotation matrix P_α^{kj} in the plane (e_k, e_j), $k \in \{1,2\}; j \in \{3,4,\ldots,n\}$.
If $k = 1$ **then** $r = 2$ **else** $r = 1$;

$$N = 2 \sum_{\ell=1,\ell\neq r}^{n} a_{k\ell}a_{j\ell} - 2a_{rk}a_{rj};$$

$$D = \sum_{\ell=1,\ell\neq r}^{n} (a_{j\ell}^2 - a_{k\ell}^2) + (a_{rk}^2 - a_{rj}^2);\ t = \frac{N}{D};$$

If $D > 0$ **then** $\alpha = 1/2\,\mathrm{arctg}\, t$;
If $D < 0$ **then** $\alpha = 1/2\,\mathrm{arctg}\, t + \pi/2$;
If $D = 0$ **and** $N > 0$ **then** $\alpha = \pi/4$;
If $D = 0$ **and** $N < 0$ **then** $\alpha = -\pi/4$;
$\tilde{A} = P_\alpha^{kj} A \left(P_\alpha^{kj}\right)^\top$;
Set $A = \tilde{A}$;

begin;

1. $\tilde{\beta}_{\mathbb{R}}(A) = \sqrt{a_{11}^2 + \sum_{i=3}^{n} a_{1i}^2 + \sum_{i=2}^{n} a_{2i}^2}$;

2. Find i, j in $\{1,\ldots,n\}$, $i \neq j$: $a_{ij}^2 + a_{ji}^2$ is maximum. Interchange the first and ith rows and the first and ith columns. Interchange the second and jth rows and the second and jth columns.
Set $P = I$.

3. Function 1. Set $P = P_\alpha^{12}P$.

4. $k = 1$; **For** $j = 3$ **to** n Procedure 2; set $P = P_\alpha^{kj}P$; Procedure 1; $P = P_\alpha^{12}P$; **end for**;

5. $k = 2$; **For** $j = 3$ **to** n Procedure 2; $P = P_\alpha^{kj}P$; Function 1; $P = P_\alpha^{12}P$; **end for**;

6. $\beta_{\mathbb{R}}(A) = \sqrt{a_{11}^2 + \sum_{i=3}^{n} a_{1i}^2 + \sum_{i=2}^{n} a_{2i}^2}$;

7. If $\tilde{\beta}_{\mathbb{R}}(A) - \beta_{\mathbb{R}}(A) < \varepsilon$ **then goto 8**
else set $\tilde{\beta}_{\mathbb{R}}(A) = \beta_{\mathbb{R}}(A)$; **goto 4**;
endif;

8. end;

V. EXAMPLES

Example 1: For the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -91 & -55 & -13 \end{bmatrix}$$

with the spectrum $\Lambda = \{-7, 3 \pm 2\mathbf{i}\}$ the stability radius approximation $\beta_{\mathbb{R}} \approx 0.45797643428764129$ has been achieved within 35 iterations of the Algorithm.

The corresponding destabilizing perturbation is presented here with 10^{-6} accuracy

$$E_* \approx \left[\begin{array}{rrr} 0.043822 & 0.024321 & -0.398275 \\ -0.007437 & 0.070759 & -0.208631 \\ -0.002016 & 0.003454 & 0.001457 \end{array} \right] .$$

The validity of the result is certified by an application of an alternative analytical approach presented in [10].

Example 2: For the matrix [6]

$$A = \left[\begin{array}{rrrrrr} -0.4 & 7 & 0 & 0 & 0 & 0 \\ -5 & -0.4 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -2 & 0 & 0 \\ 0 & 0 & 4 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 0 & 0 & -5 \end{array} \right]$$

with spectrum

$$\begin{aligned} \{-0.382305489688833 &\pm 5.80808592930962\mathbf{i}, \\ -0.936044959337319 &\pm 2.82104666731864\mathbf{i}, \\ &- 5.16329910194769, -5\}. \end{aligned}$$

the approximation $\beta_{\mathbb{R}} \approx 0.51053919404744041444$ has been achieved within 93 iterations of the Algorithm. The corresponding destabilizing perturbation is $E_* \approx$

$$\left[\begin{array}{rrrrrr} 0.355464 & -0.006308 & 0.061022 & 0.024588 & -0.001662 & -0.000019 \\ 0.009547 & 0.342690 & 0.029446 & -0.045096 & -0.006499 & 0.000051 \\ -0.086570 & 0.027549 & -0.012752 & -0.009460 & -0.000085 & 0.000009 \\ 0.019055 & 0.023281 & 0.005187 & -0.001834 & -0.000534 & 0.000003 \\ -0.000931 & 0.003292 & 0.000106 & -0.000502 & -0.000057 & 5 \cdot 10^{-7} \\ -0.000947 & 0.000405 & -0.000131 & -0.000117 & -0.000003 & 1 \cdot 10^{-7} \end{array} \right] .$$

(represented with the 10^{-6} accuracy). The spectrum of the nearest unstable matrix $A + E_*$ is

$$\{\pm 5.803736292929\,\mathbf{i}, -5.16289231052, -4.999518210930,$$
$$-0.977039086012178 \pm 2.82454550673053\mathbf{i}\} .$$

Note, that corresponding approximation for the stability radius in the 2-norm, as for 0.3612, is presented in [6].

Example 3: Let

$$\{a_j + \mathbf{i}b_j\}_{j=1}^{8} = \{-1 \pm 3\mathbf{i}, -2 \pm 7\mathbf{i},$$
$$-3 \pm 5\mathbf{i}, -4 \pm 9\mathbf{i}, -5 \pm 10\mathbf{i}, -6 \pm 11\mathbf{i}, -7 \pm 15\mathbf{i}, -8 \pm 14\mathbf{i}\} .$$

Consider the matrix $A = Q^{\top} A_{bdiag} Q \in \mathbb{R}^{16 \times 16}$. Here $Q = 1/4 H_{16}$ where $H_{16} \in \mathbb{R}^{16 \times 16}$ is the Hadamard matrix, and A_{bdiag} is the block-diagonal matrix with 8 blocks

$$\left[\begin{array}{cc} a_j & b_j \\ -b_j & a_j \end{array} \right]$$

in the main diagonal.

For this matrix, one has $\beta_{\mathbb{R}}(A) = \sqrt{2}$ with the nearest matrix in the manifold (5) possessing eigenvalues $\pm 3\mathbf{i}$ and twelve other coinciding with those of A. In this example, application of the Algorithm is subject to an extra correction due to the existence of several pairs of rows and columns with identical values for $a_{ij}^2 + a_{ji}^2$, namely 171.125. One has to perform the procedure of the Algorithm for each such a pair and then to select the minimal value. The approximation $\beta_{\mathbb{R}} \approx 1.4142135623730951455$ is achieved within 428 iterations of the Algorithm.

VI. Conclusions

We present a new iterative method for the calculation of the real stability radius in Frobenius norm using a sequence of Givens rotations that decrease the norm of desired perturbation at every step. The advantage of the method is that, instead of solving singular value problems and systems of linear equations, we use only matrix multiplication for special rotation matrices. Moreover, the appropriate initial guess that is necessary for the up-to-date iterative methods is not required. Thus, the most complicated problem of almost all iterative methods is removed. In addition, the algorithm is very simple to implement. It seems that for large sparse matrices our approach has significant potential.

Matrix multiplication with computational complexity $\mathcal{O}(n^3)$ is a central operation of the presented algorithm. Optimization of this operation for Givens matrices is the subject of future research.

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