Local Restoration of Images Distorted by Rician Noise

David Asatryan
Institute for Informatics and Automation Problems of the NAS RA,
Russian-Armenian University
Yerevan, Armenia
e-mail: dasat@iiap.sci.am

Liana Andreasyan
National Polytechnic University of Armenia,
Yerevan, Armenia
e-mail: l.andreasyan@polytechnic.am

Grigor Sazhumyan
Institute for Informatics and Automation Problems of the NAS RA,
Yerevan, Armenia
e-mail: grigorsazhumyan@gmail.com

***Abstract*—The problem of restoring an image distorted by the Rice model is considered. The restoration algorithm is based on estimating the parameters of the Rice distribution from the sample and correcting distorted pixels accordingly. The solution to the problem is based on the concept of using a restricted local neighbourhood (RLN) of the recovered pixel, which is known in the literature. It is proposed to test the statistical hypothesis of conformity to the Rice distribution for each selected RLN. A method for selecting RLNs with sufficiently large signal-to-noise ratios is proposed. The results of the experimental study are given.**

***Keywords*—Rice distribution, restricted local neighbourhood, signal-to-noise ratio, maximum-likelihood estimation.**

I. INTRODUCTION

The Rice distribution describes a wide range of problems in which the output signal is formed as a sum of the initial deterministic signal and Gaussian noise, and the analysed quantity is the amplitude of the resulting signal [1]. After the publication of Rice's paper and up to the present time, this distribution has been used as a mathematical model in the study of magnetic resonance imaging problems, radio signal reception and processing, sound echo analysis, and many others [2, 4]. In these problems, it is often required to suitably separate the components from which the signal is formed. The main task in this case is a sufficiently accurate estimation of the parameters of the Rice distribution from a sample of the measured signal and determination of the parameters of the deterministic signal and noise according to these estimations. In this case, an important characteristic is the signal-to-noise ratio SNR [3].

There is an extensive literature devoted to various useful properties and applications of the distribution to questions of investigating the accuracy of statistical estimation of the parameters of the Rice distribution [2]. Estimates based mainly on the maximum likelihood method [4, 5] and the method of moments [6] are proposed. The computational aspects of the proposed estimators are also discussed. Thus, in [7, 8], a comparative analysis of different methods is carried out, and the advantages and disadvantages of these methods are experimentally shown.

Immediately after the appearance of Rice's work, noise reduction methods for magnetic resonance (MR) images that are Rice distributed began to receive special attention in the literature. Noise in magnetic resonance images is usually modelled using a Rice distribution formed by the superposition of uncorrelated Gaussian noise with zero mean and constant variance in both the real and imaginary parts of complex data [9].

There are survey works containing a thorough review of the mentioned methods, notes, for example, the works [10-18]. In [13], a method for correcting the bias of the signal parameter estimation is also proposed.

An analysis of noise reduction methods proposed in the scientific literature allows us to classify these methods into two classes: non-local and local.

Non-local methods process the whole image, involving spectral analysis apparatus, differential analysis apparatus, various filtering methods, and others. It is noted that the use of conventional filtering methods to suppress noise in an image is inappropriate because they usually lead to blurred edges and loss of fine structures.

Local methods deal with a specific part of the image based on the available information about the expected type of distortion factor in that area.

In this paper, we propose a distorted image restoration procedure based on the technique proposed in [15] with some improvements. As a solution to the problem, this paper proposes the concept of using a restricted local neighbourhood where the true intensity for each noisy pixel is estimated from a set of selected neighbouring pixels. In this paper, a method for denoising the magnitude of magnetic resonance (MR) images that are Rice distributed is proposed. A scheme is developed to locally select a suitable subset of pixels from which the baseline signal is estimated.

According to the mentioned concept, the estimated intensity of the centre pixel is estimated from the set of selected pixels, the value of which is inserted instead of the measured value if necessary. At the same time, some pixels of the original image that do not fulfil certain conditions remain unchanged. Therefore, we call the described procedure a local or partial image restoration procedure. Other methods of partial image restoration can be found in the literature.

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This paper proposes a procedure for the partial restoration of an image distorted by the Rice model. An experimental study of the proposed technique is carried out on artificial examples, applying various known methods of image quality comparison.

II. Research Methodology

We will consider a Grayscale 8-bit grayscale image A(i, j) i = 1, 2, ..., N; j = 1, 2, ..., M, where $A(i,j) \in \underline{0,255}$. According to the above concept, the notion of a restricted local neighbourhood (RLN) is introduced, implying the selection of a compact subset of pixels that are in the neighbourhoods of each other. It is more convenient to choose a rectangular region with dimensions kxk, k = 2t + 1, t = 2, 3, ..., t_{max}. The whole image is processed by sliding scanning, and the data of each region is processed by a certain method. Then the obtained result is assigned to the centre pixel of the RLN.

It should be noted that despite the accepted model of Rice distribution for the whole image, the distributions of pixel samples from individual RLNs may deviate from the Rice type, so for each selected RLN, the statistical hypothesis of conformity to the Rice distribution is tested. A method of selecting RLNs with sufficiently large values of signal-to-noise ratio (SNR) is proposed.

The restoration algorithm consists of the following steps:

Step 1: The image with NxM pixels by sliding scanning with step 1 is divided into square type RLNs with dimensions kxk, where k = 2t + 1, t = 2, 3, ..., t_{max}. For restoration, select the central pixels of the RLNs, i.e. pixels with indices i = n + t, n + t + 1, ..., N - t; j = m + t, m + t + 1, ..., M - t; n = 1, 2, ..., N; m = 1, 2, ..., M. The total number of RLNs under consideration is equal to N1xM1, where N1 = N - 2t, ..., M = M - 2t.

Step 2: For each RLN, estimate SNR as the ratio of mean intensity to RMS value across the pixels of the RLN. We denote the set of RLNs for which SNR > γ_0 by V_0. It is recommended to choose $\gamma_0 \geq 3$.

Step 3: For each selected RLN, test the statistical hypothesis that the sample formed from the intensities of all RLN pixels corresponds to the Rice distribution. The test should be performed using the Shapiro-Wilks statistic [3].

Note: It is not reasonable to use chi-square, Kolmogorov-Smirnov, etc., statistics, as they are acceptable for sufficiently large sample sizes [19].

Step 4: If the distribution hypothesis is confirmed, the distribution parameters ν and σ are estimated using one of the methods described in [7, 8]. The maximum likelihood method is preferred. Next, the centre pixel of the original RLN is replaced by the value $\hat{\nu} = \sqrt{|M^2 - \tau^2|}$, where M and τ are the current values of the estimation of the parameters ν and σ of the Rice distribution.

Note: The formula $\hat{\nu} = \sqrt{|M^2 - \tau^2|}$ represents the bias of the ν parameter estimate with some approximation [12].

Step 5: Compare the newly formed image with the original image using the two image similarity criteria, PSNR and W^2.

III. Experimental Results

The experiments were performed using a Python environment, in classes of images of two types.

The first type was formed by generating certain images, each pixel of which was taken as a deterministic component in the Rice distribution model, and the random component had a variance fixed for the whole image. The second class contained images noisy with additive white noise, as well as images from different databases distorted by noise of certain types.

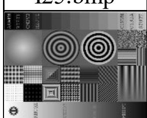
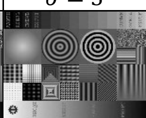
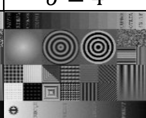
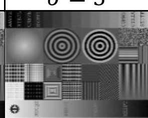
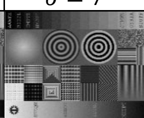
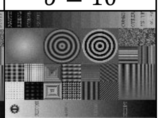
In this paper, a ready-to-use set of noisy images from the well-known TID2013 database is selected [20].

Experiment 1: To verify the effectiveness of the restoration procedure, a model of the noisy image using the Rician model was used.

Table 1 shows the images obtained from the simulation at values σ = 3, 4, 5, 7, 10, respectively, as well as the PSNR values between the original and distorted images (PSNR-OD), between the distorted and restored images (PSNR-DR). The corresponding similarity scores of W^2-OD and W^2-DR images are given for comparison.

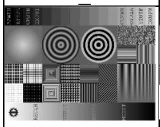
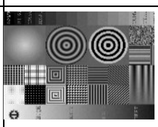
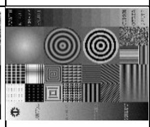
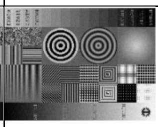
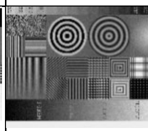
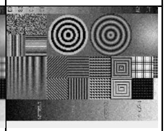
The given data confirms the effectiveness of the applied approach and the proposed restoration algorithm. The quality of the reconstructed image, as assessed by both methods, is comparable to that of the distorted image.

Table 1. Simulation results of the image distorted by Rician noise and restoration

I25.bmp	$\sigma = 3$	$\sigma = 4$	$\sigma = 5$	$\sigma = 7$	$\sigma = 10$
					
PSNR-OD	38.06	35.21	31.72	28.49	24.66
PSNR-DR	38.32	35.66	32.01	28.80	24.87
W^2-OD	0.96	0.94	0.89	0.83	0.74
W^2-DR	0.96	0.94	0.89	0.83	0.74

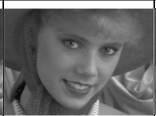
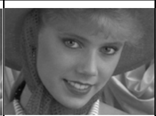
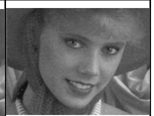
Experiment 2: The image selected for the experiment is noisy with additive Gaussian noise. The I25_01 image processed in Experiment 1 is chosen to compare the two applied denoising methods. The results in Table 2 also show the improvement of the image quality during restoration.

Table 2. Restoration results of I25_01 image noised with Gaussian noise

I25_01	I25.01.1	I25.01.2	I25.01.3	I25.01.4	I25.01.5
					
MOS	5.1429	4.6765	4.400	3.8857	3.4444
PSNR-OD	32.09	30.96	29.19	27.00	24.39
PSNR-OR	32.37	31.48	29.95	27.92	25.34
W^2-OD	0.92	0.91	0.89	0.86	0.83
W^2-OR	0.92	0.91	0.89	0.87	0.83

Experiment 3: It is of interest to apply the proposed restoration procedure to images noisy by different types of distortion algorithms. Table 3 summarises the results of the I04_01 image processing. In this example, cases of both improvement and deterioration of the restoration quality are observed. The reason for this phenomenon can be explained by comparing the quality evaluation results of different methods: they give opposite results. In the case of weak distortions, the recovery procedure does not seem to be able to adequately estimate the required parameters, so the PSNR recovery quality index deteriorates, while the W^2 index responds to more stable structural properties. In case of strong distortions, the opposite process occurs: the parameters of the restoration algorithm are estimated more accurately, while the structural properties are distorted to a greater extent, so W^2 decreases.

Table 3. Restoration results of noised I04_01 images

I04.01	I04.01.1	I04.01.2	I04.01.3	I04.01.4	I04.01.5
					
MOS	5.97143	5.75676	5.27027	5.24324	4.71429
PSNR-OD	30.14	29.26	27.85	25.95	23.61
PSNR-OR	29.87	29.10	27.87	26.11	23.74
W^2-OD	0.73	0.73	0.74	0.74	0.76
W^2-OR	0.73	0.73	0.72	0.71	0.70

IV. Conclusion

The problem of restoring an image distorted by the Rice model is considered. The restoration algorithm is based on the estimation of parameters of the Rice distribution over the sample and the correction of distorted pixels according to them. To check the efficiency of the restoration procedure, we simulated image noise according to the Rice model and also used image noise with white noise from the TID2013 database. The analysis of the given results allows us to draw the following conclusions.

- Models based on the Rice distribution allow not only to separate deterministic and random components from the summed signal, but also to recover the distorted region in multi-parameter objects like a digital image.
- The procedure of partial restoration of distorted images in the presence of preliminary information about the nature of the distortion can be considered as an alternative to the known filtering methods, usually applied to the whole image.
- The quality of the restored image should be evaluated using not only the PSNR criterion but also the structural proximity measure W^2.
- The measure W^2 characterises not only the degree of similarity of particular images, but also the stability of conclusions regarding the quality of the restoration procedure.
- The proposed methodology and software tools can be applied to other types and models of image noise.

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